(NASA-TM-X-71882) ON THE RELATION OF J SUB I TO WORK DONE PER UNIT UNCRACKED AREA: TOTAL, OR COMPONENT 'DUE TO CRACK' (NASA) 9 p HC $3.50 CSCL 13M

N76-18533

Unclas
G3/39 18503

NASA TECHNICAL MEMORANDUM

NASA TM X-71882

NASA TM X-71882

ON THE RELATION OF J_I TO WORK DONE PER UNIT UNCRACKED AREA -- TOTAL, OR COMPONENT "DUE TO CRACK"

by John E. Srawley
Lewis Research Center
Cleveland, Ohio

TECHNICAL PAPER to be presented at
Committee E-24 of the American Society for
Testing and Materials
Orlando, Florida, March 22-26, 1976

ON THE RELATION OF J_I TO WORK DONE PER UNIT UNCRACKED AREA -- TOTAL, OR COMPONENT "DUE TO CRACK"

by John E. Srawley

Lewis Research Center

E-8655

The direct evaluation of Rice's J_I [1, 2] as a function of displacement, from the load-displacement record of a test of a single precracked specimen, has been discussed by several authors [3, 4, 5, 6]. However, these authors dealt with certain particular cases, not with the general form of the underlying relation, which is shown later to be as follows:

$$J_I = \frac{\partial \ln U}{\partial \ln (W - a)} \cdot \frac{U}{B (W - a)} = \frac{\varphi U}{B (W - a)} \tag{1}$$

where U is the work done on a quasiplanar specimen of uniform thickness, B, width (or depth), W, and fixed uniform crack length, a, figure 1, when the displacement of the load is increased from zero to some given value. The same form of relation holds if U and φ are calculated in terms of the component of the displacement that is "due to the crack" [3, 4] rather than the total displacement, as is also shown later. Obviously, in general, U_a is not the same as U_t, nor is φ_a the same as φ_t, where the subscripts 'a' and 't' denote respectively, 'due to the crack' and 'total'.

The significance of (1) depends on the ability to estimate values of the logarithmic derivative, φ, with practical accuracy for the ideal extremes of material behavior: linear-elastic, and rigid/perfectly plastic, figure 2. If the value of φ should happen to be practically the same for both extremes, then it is reasonable to suppose that it might be the same for intermediate, real material behaviour, and to test this working hypothesis by judicious experiment. Should the value of φ indeed be practically independent of material behaviour, then the evaluation of

J_I as a function of displacement reduces to a set of measurements of areas under a single load-displacement curve from a test during which the crack length remains unchanged. (It should be noted that the determination of a material property, J_{Ic}, involves further issues which are not addressed here.)

A good illustrative example is the three-point-loaded bend specimen described in ASTM Method of Test E 399-74 for Plane Strain Fracture Toughness of Metallic Materials. This specimen corresponds to figure 1(b) with $s/W = 4$ and $c/W = 0$. For rigid/perfectly-plastic material behaviour, with a/W not less than 0.3, it has been shown that the ratio of bending moment to net section modulus is a fixed multiple of the flow stress [7, 8]. From this it follows that the limit load, Q, and the work done, $U = Qg$, where g is the displacement of Q, are proportional to $(1 - a/W)^2$; then, by definition (1), $\varphi = 2$. In this kind of ideal material behaviour there is no distinction between g_a and g_t, and therefore none between U_a and U_t, or between φ_a and φ_t.

For linear-elastic material behaviour, $U_t = Qg_t/2 = g_t^2/2(g/Q)_t$. Therefore, at fixed g_t, by definition (1):

$$\varphi_t = \partial \ln (g/Q)_t^{-1}/\partial \ln (1 - a/W) = (1 - a/W)\, \partial \ln (g/Q)_t/\partial (a/W)$$

$$= (1 - a/W)\, \partial \ln (E'Bg/Q)_t/\partial (a/W) = \frac{(1 - a/W)}{(E'Bg/Q)_t}\, \partial (E'Bg/Q)_t/\partial (a/W) \qquad (2)$$

where $E' = E$ for plane stress, or $E/(1 - \nu^2)$ for plane strain, E is Young's modulus and ν is Poisson's ratio. A similar relation holds between φ_a and $(E'Bg/Q)_a$, which is equal to $(E'Bg/Q)_t - (E'Bg/Q)_0$, where $(E'Bg/Q)_0$ is the compliance coefficient for $a/W = 0$. The value of $(E'Bg/Q)_0$ was calculated to be 20, which includes a shear component. To obtain $(E'Bg/Q)_a$ as a function of a/W an accurate interpolation function [9] was used to calculate the integral of

the squared stress intensity coefficient (K^2B^2W/Q^2) between the limits 0 and a/W for a suitable set of values of a/W. This definite integral is equal to $(E'Bg/Q)_a/2$. From these results, through (2), the values of φ_t and φ_a were obtained which are shown in figure 3. For a/W greater than about 0.5, the value of φ_t is practically constant, within the range 2.02 ± 0.02, which is practically the same as the value of 2 for rigid/perfectly-plastic material behaviour. Over a wider range the following interpolation expression fits within ± 2 percent for a/W greater than 0.05:

$$\varphi_t = 2 - (0.3 - 0.7\, a/W)(1 - a/W) - \exp.\ (0.5 - 7\, a/W) \tag{3}$$

For practical purposes φ_t can be taken as 2.00 for both linear-elastic and rigid/perfectly-plastic behaviour when a/W exceeds 0.5. The experience of ASTM Committee E-24 Task Group E-24.01.09 on Elasto-Plastic Fracture Criteria strongly, but indirectly, supports the supposition that the same approximation holds for real elastic/strain-hardening-plastic materials; however, there remains a need for systematic, direct experimental confirmation.

It is apparent from figure 3 that φ_a for linear-elastic behaviour could only be taken as practically equal to 2 when a/W exceeds about 0.9. Consequently, calculations in terms of quantities "due to the crack" are of little practical value for this particular specimen.

Derivation of Equation (1)

With reference to figure 1(a), at any fixed total displacement, f_t, of one load point relative to the other, the Rice path-independent integral form of J_I [1], when the contour is taken around the specimen boundary including the crack sides except for the tip, reduces to:

$$J_I B = -\frac{\partial}{\partial a}\int_0^{f_t} P\, df_t = -\partial U_t/\partial a \tag{4}$$

$$= \partial U_t/\partial (W - a) \tag{5}$$

where W is a fixed parameter of the test, and the crack length, a, is a virtual variable. The ratio of J_I to the total work done per unit area of net cross-section is then:

$$\frac{J_I}{U_t/B\,(W-a)} = \frac{(W-a)\,\partial U_t}{U_t\,\partial\,(W-a)} = \frac{\partial \ln U_t}{\partial \ln (W-a)} \equiv \varphi_t \tag{1.1}$$

Clearly a similar relation holds for the specimen loaded by couples in figure 1(b), provided that the rotation of each couple, arctan $(2g_t/s)$, differs negligably from the ratio $2g_t/s$.

The total displacement can be treated as the sum of two parts: $f_t = f_a + f_0$, where f_0 is the displacement which a crack-free but otherwise identical body would suffer under the same force, and f_a is the additional displacement which results from the reduction in specimen stiffness due to the crack. It follows that the work done can be separated into two parts:

$$U_t = \int_0^{f_t} P\, df_t = Pf_t - \int_0^{P} f_t\, dP$$

$$= Pf_0 + Pf_a - \int_0^{P} f_0\, dP - \int_0^{P} f_a\, dP$$

$$= \int_0^{f_0} P\, df_0 + \int_0^{f_a} P\, df_a = U_0 + U_a \tag{6}$$

Since f_0, and therefore U_0 are independent of a, it follows that (1) holds in terms of U_a and φ_a as well as in terms of U_t and φ_t.

REFERENCES

1. Rice, J. R., Journal of Applied Mechanics, ASME Transactions, Vol. 35, Ser. E, No. 2, June 1968, pp. 379-386.

2. Rice, J. R., Treatise on Fracture, Vol. 2, Ch. 3, H. Liebowitz, Ed., Academic Press, New York, 1968, pp. 191-311.

3. Bucci, R. J., et al, National Symposium on Fracture Mechanics, 5th, Pt. 2, STP 514, American Society for Testing and Materials, Philadelphia, 1972, pp. 40-69.

4. Rice, J. R., Paris, P. C., and Merkle, J. G., Progress in Flaw Growth and Fracture Toughness Testing; Proceedings of the Sixth National Symposium on Fracture Mechanics, STP 536, American Society for Testing and Materials, Philadelphia, 1972, pp. 231-245.

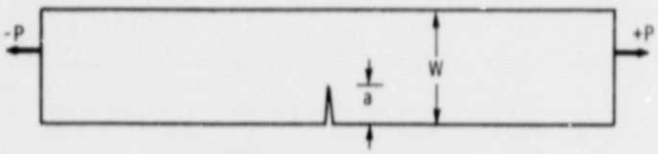
5. Begley, J. A. and Landes, J. D., Progress in Flaw Growth and Fracture Toughness Testing; Proceedings of the Sixth National Symposium on Fracture Mechanics, STP 536, American Society for Testing and Materials, Philadelphia, 1972, pp. 246-263.

6. Merkle, J. G., Progress in Flaw Growth and Fracture Toughness Testing; Proceedings of the Sixth National Symposium on Fracture Mechanics, STP 536, American Society for Testing and Materials, Philadelphia, 1972, pp. 264-280.

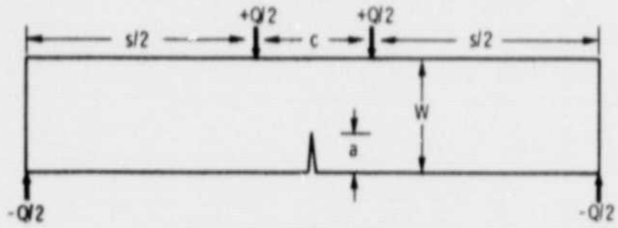
7. Green, A. P. and Hundy, B. B., Journal of the Mechanics and Physics of Solids, Vol. 4, No. 2, 1956, pp. 128-144.

8. McClintock, F. A., Welding Journal Research Supplement, Vol. 40, No. 5, May 1961, pp. 202-209.

9. Srawley, J. E., Wide Range Stress Intensity Factor Expressions for ASTM E 399 Standard Specimens, International Journal of Fracture Mechanics (to be published).



(a) SPECIMEN LOADED BY NORMAL FORCES.



(b) SPECIMEN LOADED BY COUPLES.

Figure 1.

E-8655

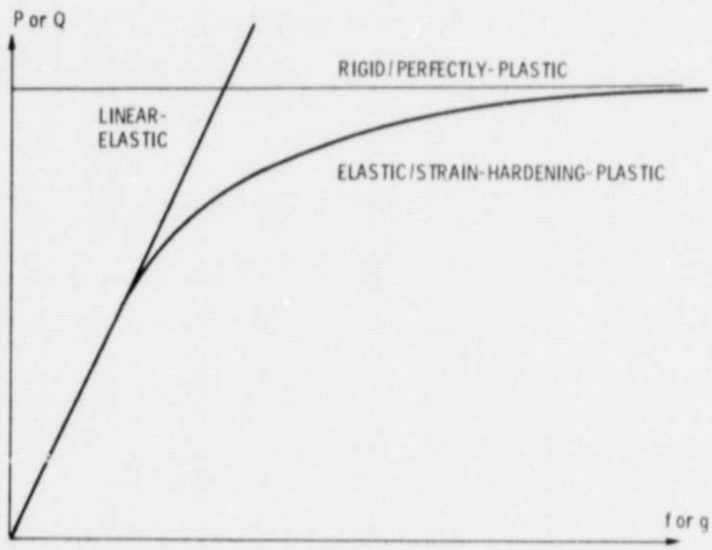


Figure 2. - Test record of force versus relative displacement.

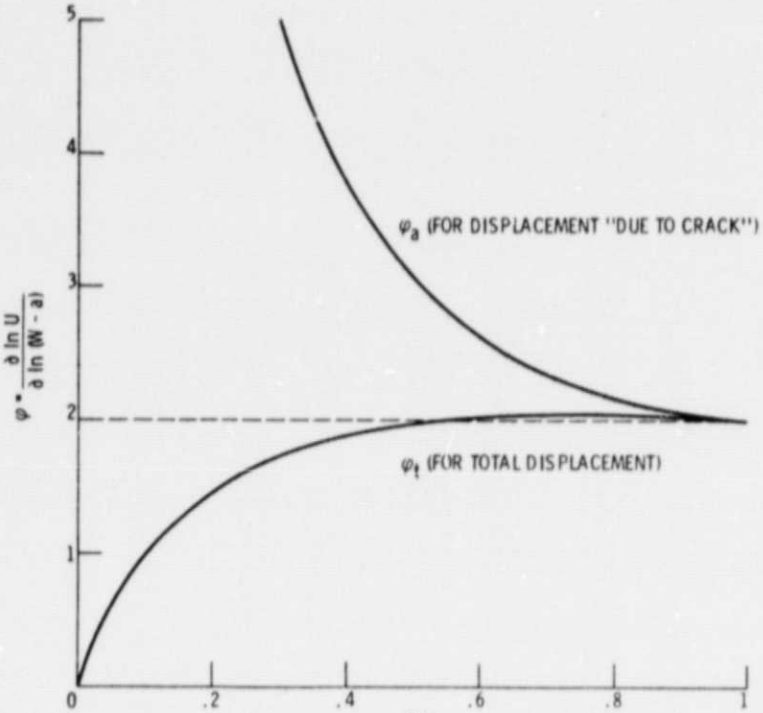


Figure 3. - Values of φ, the ratio of J_I to work done per unit area of uncracked cross-section, for an ASTM E-399 three-point bend specimen of a linear-elastic material.

E-8655

NASA-Lewis